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The Rebellion Number in Graphs

V. Mohanaselvi and P. Shyamala Anto Mary

***Abstract* -** A set $R \subseteq V$ of a graph $G = (V, E)$ is said to be a 'rebellion set' of G, if $|N_R(v)| \leq |N_{V\setminus R}(v)|$, $v \in R$ and $|R| \geq |V\setminus R|$. The rebellion number rb (G) is the minimum cardinality of any rebellion set in G. In this paper, we defined rebellion number, strong rebellion number, global rebellion number, total rebellion number for simple graph. Also, we determined its tight bounds for some standard graph and characterize these parameters.

Index Terms - Defensive alliance, global defensive alliance, global rebellion number, rebellion number, strong defensive alliance, strong rebellion number, and total rebellion number.

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1. INTRODUCTION

ALL the terms defined here are used in the sense of Harary.

A graph is a finite non-empty set of objects called vertices or nodes together with a set of unordered pairs of distinct vertices of *G*, called edges or lines.The word alliance means a bound or connection between individuals, families, states or parties. The union of individuals in an alliance is thought of to be stronger than the sole individual. Alliance in graphs was first introduced by Kristiansen, Hedetniemi and Hedetniemi.

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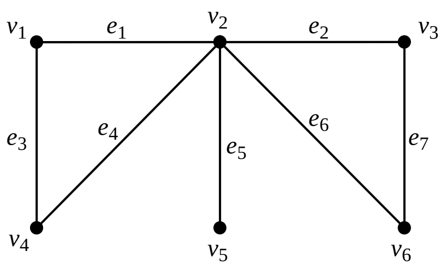
Suppose nations at war, the individual nations are represented as vertices and the relations between them are represented as edges [4]. In [5], the global (strong) defensive alliance and its number were introduced and their bounds are studied. The alliance numbers for planar graphs are introduced in [6]. The alliance numbers for planar graphs are introduced in [6]. The domination, accurate domination and accurate total domination for simple graph are refereed from [1], suppose nations at war, the individual nations are represented as vertices and the relations between them are represented as edges [4]. In [5], the global (strong) defensive alliance and its number were introduced and their bounds are studied. The alliance numbers for planar graphs are introduced in [6]. The domination, accurate domination and accurate total domination for simple graph are refereed from [1], [3]. A non-empty set of vertices, $S_{da} \subseteq V$ is called a defensive alliance if for every vertex $V \in S$, $|N[v] \cap S| \geq |N(v) \cap (V\setminus S)|$. The defensive alliance number is the minimum cardinality of all defensive alliance sets of G and it is also a dominating set of G. A set $S \subseteq V$ is said to be a dominating set in G, if every vertex in $V\setminus S$ is adjacent to some vertex in S. The domination number of G is the minimum cardinality taken overall dominating sets in G and is denoted by γ (G). A dominating set with cardinality γ (G) is denoted by γ (G) – set. A total dominating set D of G is a dominating set such that the induced subgraph <D> has no isolated vertices. The total domination number of G is the minimum cardinality of a total dominating set of G. In this paper, we have introduced the rebellion set, strong rebellion set, global rebellion set, total rebellion set by interchanging the inequality in the alliance set. Also, we determined its tight bounds for some standard graph and characterize this parameters.

2. Main Results

Definition 2.1

A set $R \subseteq V$ of a graph $G = (V, E)$ is said to be a 'rebellion set' (reb set) of G, if $|N_R(v)| \leq |N_{V\setminus R}(v)|$, $v \in R$ and $|R| \geq |V\setminus R|$. The rebellion number rb (G) is the minimum cardinality of any rebellion set in G.A rebellion set with cardinality rb (G) is denoted by

Example 2.2



G

figure 2.1

rb – set=$\{v_1,v_2,v_6\}$, rb(G)=3

Definition 2.3

A set $R \subseteq V$ of a graph $G = (V, E)$ is said to be a 'strong rebellion set'(srb– set) of G, if $|N_R(v)| \leq |N_{V\setminus R}(v)|$, $v \in R$ and $|R| \geq |V\setminus R|$. The strong rebellion number rb_s (G) is the minimum cardinality of any strong rebellion set in G. A strong rebellion set with cardinality rb_s (G) is denoted by rb_s (G)–set.

Example 2.4

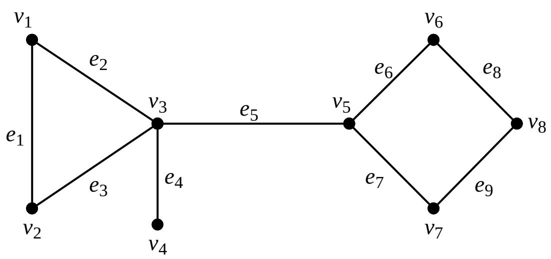


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Figure 2.2

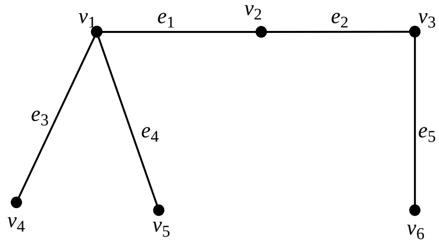
rb_s– set=$\{v_1,v_4,v_5,v_8\}$, rb_s(G)=4

Definition 2.5

A rebellion set R of a graph G said to be global rebellion set (grb – set), if R is a dominating set of G.

The global rebellion number rb_g (G) is the minimum cardinality of any global rebellion set in G.A global rebellion set with cardinality rb_g(G) is denoted by rb_g(G)– set.

Example 2.6



G

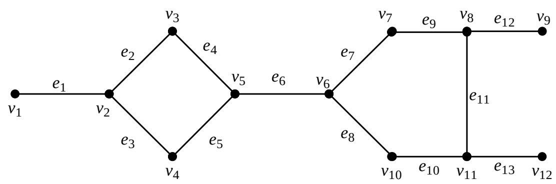
figure 2.3

rb_g–set=$\{v_3,v_4,v_5\}$, rb_g(G)=3

Definition 2.7

A set $R \subseteq V$ of a graph $G = (V,E)$ is said to be a 'total rebellion set'(trb– set) of G, if $|N_R(v)| \leq |N_{V\setminus R}(v)|$, $v \in R$ and $|R| \geq |V\setminus R|$. The total rebellion number rb_t (G) is the minimum cardinality of any total rebellion set in G.A total rebellion set with cardinality rb_t (G) is denoted by rb_t(G)–set.

Example 2.8



G

figure 2.4

rb_t– set=$\{v_2,v_3,v_6,v_7,v_{11},v_{12}\}$,

srb_t(G)= 6

Theorem 2.9

A rb_g– set is a γ -set of G if and only iff G is isomarphic to P_2 (or) K_2.

Remark

In general a dominating set need not be a rb_g-set.

For example ,

1.

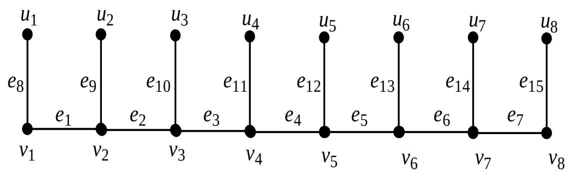


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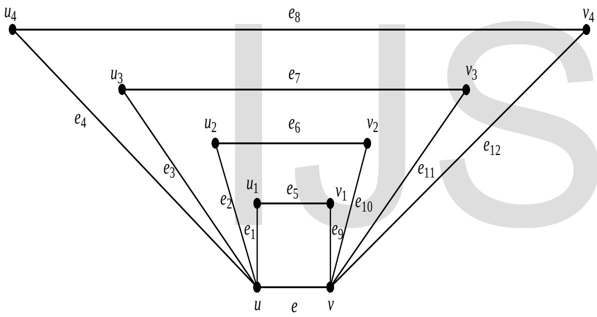
figure 2.5

In this figure 2.5 ,

the reb– set R={u_1,u_2,u_3,u_4,u_5,u_6,u_7,u_8} is a rb– set of G. Hence rb (G) = 8.

Also, γ – set = {u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8} is a set of G and hence γ (G)=8.Here the reb– set is a γ – set of G and γ – set is the reb – set of G.

2.



B_4

Figure 2.6

In this figure 2.6, the reb– set R={u,v_1,v_2,v_3,v_4} is a rb – set of G. Hence rb (G) =5.

Also, γ – set = {u, v} is a set of G and hence

γ (G)=2. Here a γ – set is not the reb– set of G.

3.

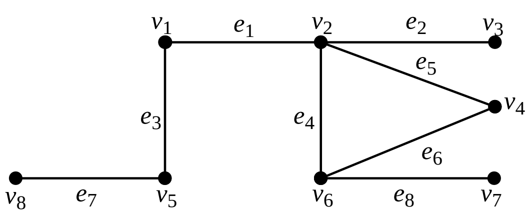


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Figure 2.7

In this figure 2.7, the reb– set R= {v_3, v_4, v_7, v_8} is a rb– set of G. Hence rb (G) =4.

Also, γ – set = {v_2, v_5, v_6} is a set of G and hence γ (G) = 3.Here the reb– set is not a γ –set of G.

Theorem 2.10

For any graph , $\gamma(G) \leq rb_g(G)$.

Proof :

Since every grb– set of G is a γ– set of G and hence the result.

For example , the equality holds for P_2 ,C_4.

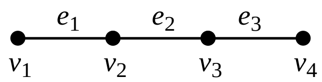

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Figure 2.8

$\gamma(G)=2$, $rb_g(G)=2$

Theorem 2.11

If G is a non– complete graph with P≥5, then the induced sub graph of a reb – set R disconnected.

Proof :

Let G be a non– complete graph with five vertices. Suppose R is connected with minimum number of three vertices say{v_1,v_2,v_3}.Then every vertices in R not satisfy the condition $|N_R(v)| \leq |N_{V\setminus R}(v)|$ which contradiction , R is a rebellion set. Hence R must be disconnected.

Theorem 2.12

If G is a non – acyclic graph with odd number of vertices , then the rebellion Set R is not strong.

Proof :

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Let G be an odd order cyclic graph and R be rebellion set of G. Take P=2r+1.Then R must contain r+1 vertices. Also, there exist at least one $|N_R(v)| \leq |N_{V\backslash R}(v)|$, $v \in R$. which contradiction ,R is a rebellion set. Hence the proof.

Theorem 2.13

Every strong rebellion set of G is a global rebellion set.

Proof:

Let rb_s be a strong rebellion set of G. Suppose rb_g is not a global rebellion set. Then rb_s is not a dominating set of G. That is, there exist $u \in V \setminus rb_s$ and $v \in rb_s$ such that u & v are not adjacent.

It gives $|N\ rb_s(v)| \leq |N_{V\setminus rbs}(v)|$ which contradiction. rb_s– set is a rb_g– set of (G).

Hence the proof.

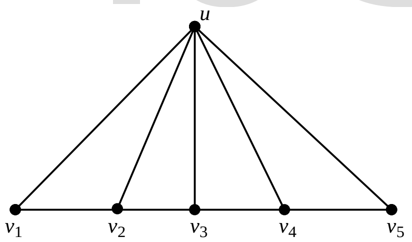

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Figure 2.9

rb_s– set $=\{v_1,v_3,v_5\}$ rb_g– set$=\{v_1,v_3,v_5\}$

Theorem 2.14

Let G be a graph with p vertices and without isolated vertices. Then $rb_t(G) = 0$ if and only if $G = mK_2$, $m \geq 1$.

Proof:

For m =1 the result is obvious. For m >1, suppose $G \neq K_2$, then there exists a component in G which has a vertex v adjacent to atleast two vertices u and w. This implies that V-{v} is an total rebellion set which is a contradiction. This proves the necessary part.

The sufficient part is obvious.

Theorem 2.15

For any graph G with m cut vertices, then $rb(G) \leq m+1$.

Proof:

Let S be the set of all cut vertices of G with $|S| = m$. Then there exist $u \in S$ such that {V\S U {u}} is a rb– set.

Therefore, $rb(G) \leq \left[\frac{\Delta(G)+1}{2}\right]$.

Hence, $rb(G) \leq m+1$.

Theorem 2.16

For any graph G, $rb(G) \geq \left\lceil \frac{p}{2} \right\rceil$

Proof:

Since the rebellion number rb(G) is the minimum cardinality of any rebellion set in G then fro m the condition of rebellion number we have

$rb(G) \geq \left\lceil \frac{p}{2} \right\rceil$.

Theorem 2.17

For any graph G, $rb(G) \geq \left[\frac{\Delta(G)+1}{2}\right]$

Proof:

For any graph G , we have the maximum degree $\Delta(G) \leq p-1$.

Then $\left[\frac{\Delta(G)+1}{2}\right] \leq \frac{p}{2} \leq \left\lceil \frac{p}{2} \right\rceil \leq rb(G)$.

Hence, $\left[\frac{\Delta(G)+1}{2}\right] \leq rb(G)$.

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Theorem 2.18

For the Path graph P_n , $n \geq 2$, rb(G)= $\left\lceil \frac{n}{2} \right\rceil$

Proof:

Let G be path graph P_n with atleast two vertices and R be an rebellion set in G.

Step 1: Since rebellion number rb(G) is the minimum cardinality taken overall rebellion set in G , we have rb(G) $\leq |R| = \left\lceil \frac{n}{2} \right\rceil$ … (1)

Step 2: Suppose R is a rb set of G. Then R has at least $\left\lceil \frac{n}{2} \right\rceil$ number of vertices and hence

rb(G)= $|R| \geq \left\lceil \frac{n}{2} \right\rceil$ …(2)

From (1) & (2) , the result follows.

For example,

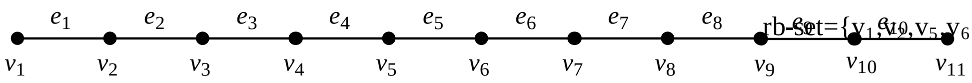

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Figure 2.10

Rb– set = {v_1, v_3, v_5, v_7, v_9, v_{11}} , rb(G)=6

Theorem 2.19

For the Wheel graph W_n , $n \geq 2$, rb(G)= $\left\lceil \frac{n}{2} \right\rceil + 1$

Proof:

Let G be Wheel graph W_n with atleast two vertices and R be a rebellion set in G.

Step 1: Since rebellion number rb (G) is the minimum cardinality taken overall rebellion set in G , we have

$$\text{rb(G)} \leq |R| = \left\lceil \frac{n}{2} \right\rceil + 1 \qquad \ldots(1)$$

Step 2: Suppose R is a rb set of G.

Then R has at least $\left\lceil \frac{n}{2} \right\rceil + 1$ number of vertices and

$$\text{hence} \quad \text{rb(G)} = |R| \geq \left\lceil \frac{n}{2} \right\rceil + 1 \qquad \ldots(2)$$

From (1) & (2) ,the result follows.

For example,

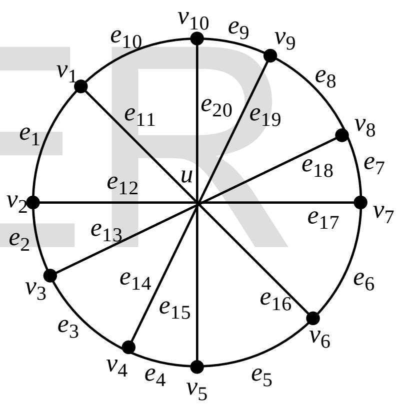

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Figure 2.11

rb-set={v_1,v_2,v_5,v_6,v_8,v_9} , rb(G)=6

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